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ON $p\delta s$ -IRRESOLUTE FUNCTIONS

ABSTRACT. The object of the present paper is to define and investigate the concepts of $p\delta s$ -irresolute functions in topological spaces. And we obtain some characterizations and fundamental properties of such functions.

KEY WORDS: α -open set, semi open set, preopen set, δ -semi open set and $p\delta s$ -irresolute function.

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1. Preliminaries

Throughout this paper, spaces always mean topological spaces and $f : X \rightarrow Y$ denotes a single valued function from a space (X, τ) into a space (Y, ν) . Let S be a subset of a space (X, τ) . The closure and the interior of S are denoted by $Cl(S)$ and $Int(S)$, respectively. We recall some known definitions and properties.

Definition 1. A subset S of a space (X, τ) is said to be α -open [16] (resp. semi open [10], preopen [12]) if $S \subset Int(Cl(Int(S)))$ (resp. $S \subset Cl(Int(S))$, $S \subset Int(Cl(S))$).

A point $x \in X$ is called the δ -cluster point of A if $A \cap Int(Cl(U)) \neq \emptyset$ for every open set U of X containing x . The set of all δ -cluster points of A is called the δ -closure [17] of A and is denoted by $Cl_\delta(A)$. A subset A of X is called δ -closed [22] if $A = Cl_\delta(A)$. The complement of a δ -closed set is called δ -open [22]. A subset A of X is said to be δ -semi open [18] if there exists a δ -open set U of X such that $U \subset A \subset Cl(U)$. The complement of a δ -semi open set is called δ -semi closed [18]. A point $x \in X$ is called the δ -semicluster point of A if $A \cap U \neq \emptyset$ for every δ -semi open set U of X containing x . The set of all δ -semicluster points of A is called the δ -semiclosure [18] of A and is denoted by $sCl_\delta(A)$.

The family of all α -open (resp. semi open, preopen, δ -semi open) sets in a space (X, τ) is denoted by $\alpha(X)$ (resp. $SO(X)$, $PO(X)$, $\delta SO(X)$). It is shown in [16] that $\alpha(X)$ is a topology for X . Moreover, $\tau \subset \alpha(X) = PO(X) \cap SO(X) \subset PO(X) \cup SO(X)$.

The complement of a preopen set is said to be preclosed [12]. The intersection of all preclosed sets in (X, τ) containing a subset A is called the preclosure [7] of A and is denoted by $pCl(A)$. The union of all δ -semi open sets of X contained in A is called the δ -semiinterior [18] of A and is denoted by $slnt_{\delta}(A)$.

2. Some properties of $p\delta s$ -irresolute functions

Definition 2. A function $f : (X, \tau) \rightarrow (Y, \nu)$ is said to be $p\delta s$ -irresolute [1] (resp. $\alpha\delta s$ -irresolute [1], semi δs -irresolute [3]) if $f^{-1}(V)$ is preopen (resp. α -open, semi open) in X for every δ -semi open subset V of Y .

Remark 1 ([1]). It is obvious that

$$p\delta s\text{-irresoluteness} \Leftarrow \alpha\delta s\text{-irresoluteness} \Rightarrow \text{semi } \delta s\text{-irresoluteness.}$$

The above implications are not reversible in general. It is shown in Examples 2.1 and 2.2 of [1] that $p\delta s$ -irresoluteness and semi δs -irresoluteness are independent of each other.

Here, we obtain the following characterizations of $p\delta s$ -irresolute functions.

Theorem 1. For a function $f : (X, \tau) \rightarrow (Y, \nu)$, the following are equivalent:

- (1) f is $p\delta s$ -irresolute,
- (2) For each $x \in X$ and each δ -semi open subset V of Y containing $f(x)$, there exists a preopen set U of X containing x such that $f(U) \subset V$,
- (3) $f^{-1}(V) \subset Int(Cl(f^{-1}(V)))$ for every δ -semi open subset V of Y ,
- (4) $f^{-1}(F)$ is preclosed in X for every δ -semi closed subset F of Y ,
- (5) $Cl(Int(f^{-1}(B))) \subset f^{-1}(sCl_{\delta}(B))$ for every subset B of Y ,
- (6) $f(Cl(Int(A))) \subset sCl_{\delta}(f(A))$ for every subset A of X .

Proof. (1) \Rightarrow (2). Let $x \in X$ and V be any δ -semi open subset of Y containing $f(x)$. By Definition 2, $f^{-1}(V)$ is preopen in X and contains x . Set $U = f^{-1}(V)$. Then by (1), the set U is a preopen subset of X containing x and $f(U) \subset V$.

(2) \Rightarrow (3). Let V be any δ -semi open subset of Y and $x \in f^{-1}(V)$. By (2), there exists a preopen set U of X containing x such that $f(U) \subset V$. Therefore, we obtain $x \in U \subset Int(Cl(U)) \subset Int(Cl(f^{-1}(V)))$ and hence $f^{-1}(V) \subset Int(Cl(f^{-1}(V)))$.

(3) \Rightarrow (4). Let F be any δ -semi closed subset of Y . Set $V = Y - F$. Then the set V is δ -semi open in Y . By (3), we have $f^{-1}(V) \subset Int(Cl(f^{-1}(V)))$ and hence $f^{-1}(F) = X - f^{-1}(Y - F) = X - f^{-1}(V)$ is preclosed in X .

(4) \Rightarrow (5). Let B be any subset of Y . Since the set $sCl_\delta(B)$ is δ -semi closed in Y , by (4) $f^{-1}(sCl_\delta(B))$ is preclosed in X and hence $Cl(Int(f^{-1}(sCl_\delta(B)))) \subset f^{-1}(sCl_\delta(B))$. Thus, we have $Cl(Int(f^{-1}(B))) \subset f^{-1}(sCl_\delta(B))$.

(5) \Rightarrow (6). Let A be any subset of X . By (5), we obtain $Cl(Int(A)) \subset Cl(Int(f^{-1}(f(A)))) \subset f^{-1}(sCl_\delta(f(A)))$. Therefore, we have $f(Cl(Int(Cl(A)))) \subset sCl_\delta(f(A))$.

(6) \Rightarrow (1). Let V be any δ -semi open subset of Y . Since $f^{-1}(Y - V) = X - f^{-1}(V)$ is a subset of X , by (6) we have $f(Cl(Int(f^{-1}(Y - V)))) \subset sCl_\delta(f(f^{-1}(Y - V))) \subset sCl_\delta(Y - V) = Y - sInt_\delta(V) = Y - V$ and hence $X - Int(Cl(f^{-1}(V))) = Cl(Int(X - f^{-1}(V))) = Cl(Int(f^{-1}(Y - V))) \subset f^{-1}(Y - V) = X - f^{-1}(V)$. Thus, we obtain $f^{-1}(V) \subset Int(Cl(f^{-1}(V)))$ and hence $f^{-1}(V)$ is preopen in X . \blacksquare

Lemma 1 ([7], [9]). *Let $\{X_\lambda : \lambda \in \Lambda\}$ be a family of spaces and U_{λ_i} be a nonempty subset of X_{λ_i} for each $i = 1, 2, \dots, n$. Then $U = \prod_{\lambda \neq \lambda_i} X_\lambda \times \prod_{i=1}^n U_{\lambda_i}$ is a nonempty preopen [7] (resp. δ -semi open [9]) subset of $\prod X_\lambda$ if and only if U_{λ_i} is preopen (resp. δ -semi open) in X_{λ_i} for each $i = 1, 2, \dots, n$.*

Theorem 2. *A function $f : X \rightarrow Y$ is $p\delta s$ -irresolute if the graph function $g : X \rightarrow X \times Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, is $p\delta s$ -irresolute.*

Proof. Let $x \in X$ and V be any δ -semi open subset of Y containing $f(x)$. Then, by Lemma 1, $X \times V$ is a δ -semi open set of $X \times Y$ containing $g(x)$. Since g is $p\delta s$ -irresolute, there exists a preopen subset U of X containing x such that $g(U) \subset X \times V$ and hence $f(U) \subset V$. Thus f is $p\delta s$ -irresolute. \blacksquare

Theorem 3. *If the product function $f : \prod X_\lambda \rightarrow \prod Y_\lambda$ is $p\delta s$ -irresolute, then $f_\lambda : X_\lambda \rightarrow Y_\lambda$ is $p\delta s$ -irresolute for each $\lambda \in \Lambda$.*

Proof. Let $\lambda_0 \in \Lambda$ be an arbitrary fixed index and V_{λ_0} be any δ -semi open subset of Y_{λ_0} . Then, by Lemma 1, $\prod Y_\gamma \times V_{\lambda_0}$ is δ -semi open in $\prod Y_\lambda$, where $\lambda_0 \neq \gamma \in \Lambda$. Since f is $p\delta s$ -irresolute, $f^{-1}(\prod Y_\gamma \times V_{\lambda_0}) = \prod X_\gamma \times f_{\lambda_0}^{-1}(V_{\lambda_0})$ is preopen in $\prod X_\lambda$ and hence $f_{\lambda_0}^{-1}(V_{\lambda_0})$ is preopen in X_{λ_0} by Lemma 1. This shows that f_{λ_0} is $p\delta s$ -irresolute. \blacksquare

Lemma 2 ([13]). *If $A \in SO(X)$ and $B \in PO(X)$, then $A \cap B \in PO(A)$.*

Theorem 4. *If $f : (X, \tau) \rightarrow (Y, \nu)$ is $p\delta s$ -irresolute and A is a semi open subset of X , then the restriction $f_{/A} : A \rightarrow Y$ is $p\delta s$ -irresolute.*

Proof. Let V be any δ -semi open subset of Y . Since f is $p\delta s$ -irresolute, $f^{-1}(V)$ is a preopen set in X . Since A is semi open in X , by Lemma 2, $(f_{/A})^{-1}(V) = A \cap f^{-1}(V)$ is preopen in A . Therefore $f_{/A}$ is $p\delta s$ -irresolute. \blacksquare

Lemma 3 ([13]). *If $A \subset B \subset X$, $A \in PO(B)$ and $B \in PO(X)$, then $A \in PO(X)$.*

Theorem 5. *Let $f : (X, \tau) \rightarrow (Y, \nu)$ be a function and $\{A_\lambda : \lambda \in \Lambda\}$ be a cover of X by preopen sets of (X, τ) . Then f is $p\delta s$ -irresolute if $f_{/A_\lambda} : A_\lambda \rightarrow Y$ is $p\delta s$ -irresolute for each $\lambda \in \Lambda$.*

Proof. Let V be any δ -semi open subset of Y . Since $f_{/A_\lambda}$ is $p\delta s$ -irresolute, $(f_{/A_\lambda})^{-1}(V)$ is preopen in A_λ . Since A_λ is preopen in X , $(f_{/A_\lambda})^{-1}(V)$ is preopen in X for each $\lambda \in \Lambda$ by Lemma 3. Thus, we have $f^{-1}(V) = X \cap f^{-1}(V) = \cup\{A_\lambda \cap f^{-1}(V) : \lambda \in \Lambda\} = \cup\{(f_{/A_\lambda})^{-1}(V) : \lambda \in \Lambda\}$ is preopen in X because the union of preopen sets is a preopen set. Therefore, f is $p\delta s$ -irresolute. ■

We recall that a function $f : (X, \tau) \rightarrow (Y, \nu)$ is said to be δ -irresolute ([17], [2]) (resp. preirresolute [21]) if $f^{-1}(V)$ is δ -semi open (resp. preopen) in X for every δ -semi open (resp. preopen) set V of Y .

Theorem 6. *Let $f : X \rightarrow Y$ be a function and $g : Y \rightarrow Z$ be a δ -irresolute function. If f is $p\delta s$ -irresolute, then the composition $gof : X \rightarrow Z$ is $p\delta s$ -irresolute.*

Proof. Let W be any δ -semi open subset of Z . Since g is δ -irresolute, $g^{-1}(W)$ is δ -semi open in Y . Since f is $p\delta s$ -irresolute, $(gof)^{-1}(W) = f^{-1}(g^{-1}(W))$ is preopen in X and hence gof is $p\delta s$ -irresolute. ■

Theorem 7. *Let $f : X \rightarrow Y$ be a preirresolute function and $g : Y \rightarrow Z$ be a function. If g is $p\delta s$ -irresolute, then the composition $gof : X \rightarrow Z$ is $p\delta s$ -irresolute.*

Proof. Let W be any δ -semi open subset of Z . Since g is $p\delta s$ -irresolute, $g^{-1}(W)$ is preopen in Y . Since f is preirresolute, $(gof)^{-1}(W) = f^{-1}(g^{-1}(W))$ is preopen in X and hence gof is $p\delta s$ -irresolute. ■

We recall that a topological space (X, τ) is said to be pre- T_2 [8] (resp. δ -semi T_2 [2]) if for any two distinct points x and y in X , there exist disjoint preopen (resp. δ -semi open) sets U and V in X such that $x \in U$ and $y \in V$.

Theorem 8. *If a function $f : (X, \tau) \rightarrow (Y, \nu)$ is a $p\delta s$ -irresolute injection and a space Y is δ -semi T_2 , then X is a pre- T_2 space.*

Proof. Let x_1 and x_2 be any two distinct points of X . Then $f(x_1) \neq f(x_2)$. Since Y is δ -semi T_2 , there exist disjoint δ -semi open sets V and W of Y containing $f(x_1)$ and $f(x_2)$, respectively. Since f is $p\delta s$ -irresolute, there exist preopen sets U and G of X containing x_1 and x_2 , respectively such that $f(U) \subset V$ and $f(G) \subset W$. It follows that $U \cap G = \emptyset$. This means that X is a pre- T_2 space. ■

Theorem 9. *If a function $f : (X, \tau) \rightarrow (Y, \nu)$ is $p\delta s$ -irresolute and a space Y is δ -semi T_2 , then a set $A = \{(x, y) : f(x) = f(y)\}$ is preclosed in $X \times X$.*

Proof. Suppose that $(x, y) \notin A$. Then $f(x) \neq f(y)$. Since Y is δ -semi T_2 , there exist disjoint δ -semi open sets V and W of Y containing $f(x)$ and $f(y)$, respectively. Since f is $p\delta s$ -irresolute, there exist preopen sets U and G of X containing x and y , respectively such that $f(U) \subset V$ and $f(G) \subset W$. Set $B = U \times G$. Then, the set B is preopen in $X \times X$ such that $(x, y) \in B$ and $A \cap B = \emptyset$. This shows that $pCl(A) \subset A$ and hence the set A is preclosed in $X \times X$. ■

Definition 3. *For a function $f : (X, \tau) \rightarrow (Y, \nu)$, the graph $G(f) = \{(x, f(x)) : x \in X\}$ is called $p\delta s$ -closed if for each $(x, y) \in (X \times Y) - G(f)$, there exist $U \in PO(X)$ containing x and $V \in \delta SO(Y)$ containing y such that $(U \times V) \cap G(f) = \emptyset$.*

Theorem 10. *If a function $f : (X, \tau) \rightarrow (Y, \nu)$ is $p\delta s$ -irresolute and Y is δ -semi T_2 , then $G(f)$ is $p\delta s$ -closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. This implies that $f(x) \neq y$. Since Y is δ -semi T_2 , there exist disjoint δ -semi open sets V and W in Y containing $f(x)$ and y , respectively. Since f is $p\delta s$ -irresolute, there exists a preopen set U of X containing x such that $f(U) \subset V$. Therefore, $f(U) \cap W = \emptyset$ and hence $(U \times W) \cap G(f) = \emptyset$. Thus, $G(f)$ is $p\delta s$ -closed in $X \times Y$. ■

Theorem 11. *If a function $f : (X, \tau) \rightarrow (Y, \nu)$ is a $p\delta s$ -irresolute injection with a $p\delta s$ -closed graph, then X is a pre- T_2 space.*

Proof. Let x and y be any distinct points of X . Then $f(x) \neq f(y)$ and hence $(x, f(y)) \in (X \times Y) - G(f)$. Since $G(f)$ is $p\delta s$ -closed, there exist $U \in PO(X)$ containing x and $V \in \delta SO(Y)$ containing $f(y)$ such that $f(U) \cap V = \emptyset$. Since f is $p\delta s$ -irresolute, there exists $G \in PO(X)$ containing y such that $f(G) \subset V$. Thus we have $f(U) \cap f(G) = \emptyset$ and hence $U \cap G = \emptyset$. This shows that X is a pre- T_2 space. ■

Definition 4. *A subset A of a topological space (X, τ) is said to be*

(1) *strongly compact relative to (X, τ) [15] if for every cover $\{V_\alpha : \alpha \in \Delta\}$ of A by preopen sets of X , there exists a finite subset Δ_0 of Δ such that $A \subset \cup_{\alpha \in \Delta_0} V_\alpha$,*

(2) *s -closed relative to (X, τ) [6] if for every cover $\{V_\alpha : \alpha \in \Delta\}$ of A by semi open sets of X , there exists a finite subset Δ_0 of Δ such that $A \subset \cup_{\alpha \in \Delta_0} sCl(V_\alpha)$,*

(3) *(X, τ) is said to be strongly compact [15] (resp. s -closed [6]) if X is strongly compact (resp. s -closed) relative to (X, τ) .*

Recall that the complement of a semi open set is said to be semi closed [5].

Lemma 4. *For a subset A of a topological space (X, τ) , the following properties hold:*

- (1) *If A is semi open, then $sCl(A)$ is semi closed and semi open,*
- (2) *If A is semi open and semi closed, then A is δ -semi open,*
- (3) *If A is semi open, then $sCl(A)$ is δ -semi open.*

Proof.(1) This follows from Proposition 2.2 of [6].

(2) This follows from Lemma 3.1 of [17].

(3) This is an immediate consequence of (1) and (2). ■

Theorem 12. *Let $f : (X, \tau) \rightarrow (Y, \nu)$ be a $p\delta s$ -irresolute function. If K is strongly compact relative to X , then $f(K)$ is s -closed relative to Y .*

Proof. Let $\{V_\alpha : \alpha \in \Delta\}$ be any cover of $f(K)$ by semi open sets of Y . Since V_α is semi open, by Lemma 4 $sCl(V_\alpha)$ is δ -semi open in Y . Since f is $p\delta s$ -irresolute, $f^{-1}(sCl(V_\alpha))$ is preopen in X for each $\alpha \in \Delta$. Since $f(K) \subset \cup_{\alpha \in \Delta} V_\alpha \subset \cup_{\alpha \in \Delta} sCl(V_\alpha)$, $K \subset f^{-1}(f(K)) \subset f^{-1}(\cup_{\alpha \in \Delta} sCl(V_\alpha)) = \cup_{\alpha \in \Delta} f^{-1}(sCl(V_\alpha))$. Since K is strongly compact relative to X , there exists a finite subset Δ_0 of Δ such that $K \subset \cup_{\alpha \in \Delta_0} f^{-1}(sCl(V_\alpha))$. Therefore, we have $f(K) \subset \cup_{\alpha \in \Delta_0} sCl(V_\alpha)$. This shows that $f(K)$ is s -closed relative to Y . ■

Corollary 1. *Let $f : (X, \tau) \rightarrow (Y, \nu)$ be a $p\delta s$ -irresolute surjection. If (X, τ) is strongly compact, then (Y, ν) is s -closed.*

Definition 5. *A topological space (X, τ) is said to be semi-connected [19] (resp. preconnected [20]) if X cannot be expressed as the disjoint union of two nonempty semi open (resp. preopen) sets.*

Theorem 13. *Let $f : (X, \tau) \rightarrow (Y, \nu)$ be a $p\delta s$ -irresolute surjection. If (X, τ) is preconnected, then (Y, ν) is semi-connected.*

Proof. Suppose that (Y, ν) is not semi-connected. Then there exist two nonempty semi open sets U and V such that $U \cap V = \emptyset$ and $U \cup V = Y$. Since U and V are semi open, by Lemma 4 $sCl(U)$ is semi open and $sCl(U) \cap V = \emptyset$. Hence $sCl(U) \cap sCl(V) = \emptyset$. Moreover, by Lemma 4, $sCl(U)$ and $sCl(V)$ are δ -semi open and $sCl(U) \cup sCl(V) = Y$. Therefore, $X = f^{-1}(sCl(U)) \cup f^{-1}(sCl(V))$ and $f^{-1}(sCl(U)) \cap f^{-1}(sCl(V)) = \emptyset$. Moreover, $f^{-1}(sCl(U))$ and $f^{-1}(sCl(V))$ are nonempty preopen sets. This shows that (X, τ) is not preconnected. This completes the proof. ■

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