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## ENHANCING THE POSSIBILITIES IN VISUALISATION OF THE FERRORESONANCE PHENOMENON

Waveforms measured and recorded in a ferroresonant circuit are the base for many computations, including simulation of ferroresonance circuit over - current/voltage responses and modelling a nonlinear coil.

During such a study of ferroresonance phenomenon, the time dependent nonlinear differential equations are derived from  $R - \Psi(i) - C$  ferroresonant circuit. The system of nonlinear equations is numerically solved using various algorithm applications. In applied mathematics, in particular the context of the nonlinear dynamical systems analysis, phase - plane/space graphs are a visual display of certain characteristics of kinds of differential equations.

The aim of the paper is a full plane/space presentation of all possible states of a test circuit when the ferroresonance occurs. Poincare maps application is also mentioned in the paper.

**KEYWORDS:** ferroresonance, Power System, ferromagnetic core coil, measurements, state space and plane representation of dynamical systems.

### 1. FERRORESONANCE PHENOMENON

Ferroresonance is an oscillating phenomenon occurring in an electric circuit equipped with a nonlinear inductance, a capacitance and also containing some resistive element. In such a circuit, any  $u - i$  condition change of any electrical component can lead to next  $u - i$  spontaneous changes at the terminals of other components [1, 2].

Power networks are especially vulnerable for ferroresonance occurrence. The dense accumulation of line and cable capacitances and transformer's magnetizing inductances creates an environment where a ferroresonance scenario cannot be excused. These nonlinear  $L -$  linear  $C$  connections are exposed to impossible to avoid power network switching events. Those events create moments of system unbalance after which a ferroresonance is ignited [3].

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Electrical equipment damages (e.g. failures in transformers, cables, and arresters and severe power quality problems) are consequences that are hard to avoid during ferroresonant overvoltage and/or overcurrent impact [4].

Therefore it is necessary to study and exam that particular phenomenon in order to learn how to predict and identify it and in the end how to protect against it. Knowledge about classification of a ferroresonance phenomenon is also required in order to choose the correct mathematical approach [5, 6].

It is worth to mention that ferroresonance as a phenomenon is so complex that it almost rejects any analytical trials of examination. The proper tools for the analysis or prediction are located in advanced computation methods, i.e. a numerical integration method [7, 8, 9].

In this paper the time-domain techniques were numerically implemented. In the first place, the ferromagnetic circuit coil with its nonlinearities and hysteresis was exam. After that the ferroresonant was intentionally ignited in test circuit and analyses have been performed. The phase plane and space visualization was provided with the use of Poincaré Maps.

The aim of performed studies was to assess and classify a ferroresonance phenomenon induced in laboratory.

## 2. CIRCUIT MEASUREMENTS

The measurements – the basis for further analysis, were performed in the test circuit composed in laboratory. This circuit is equipped with a regulated voltage source, a resistor, a nonlinear coil and a capacitor. Through an adjustment of the source voltage level one can intentionally induce a ferroresonance phenomenon in the circuit.

The industrial digital interference recorder was used to acquire the voltage and current waveforms on all circuit components. During the measurements the ferromagnetic core coil has reached its saturation region conditions.

The flux linkage is a quantity not measurable directly with the use of conventional engineering tools. For the purpose of this paper it was obtained directly from its definition as a time integral of the voltage across the coil [10]:

$$\Psi(t) = \int_{t_0}^t u_L(t) dt + \Psi(t_0) \quad (1)$$

The trapezoidal integration method is applied in numerical realization computations.

In Fig. 1 and Fig. 2 pairs of waveforms at the moment when ferroresonance occurs are presented. The measured circuit current is paired with computed flux linkage (Fig. 1). The collected measured voltages of the coil and capacitor are depicted in Fig. 2.

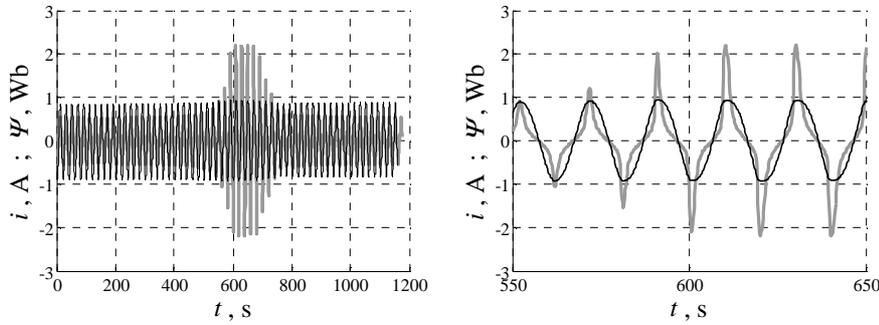


Fig. 1. Measured waveforms of circuit current (gray) and computed nonlinear coil magnetic flux (black) during ferroresonance (the scale is common for both quantities)

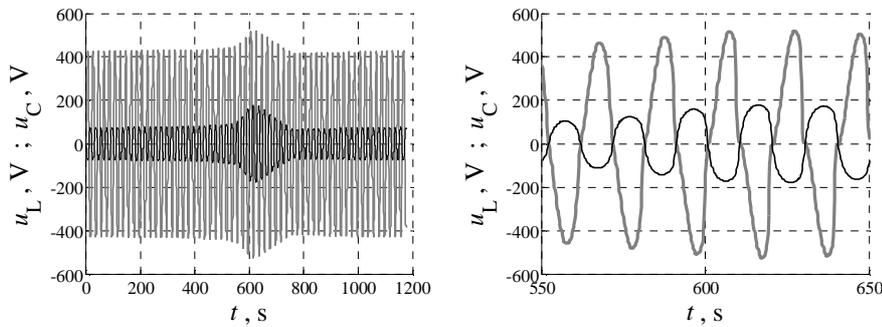


Fig. 2. Nonlinear coil (gray) and linear capacitor (black) measured voltage waveforms when ferroresonance occurs (the scale is common for both quantities)

### 3. IRON CORE COIL EXAMINATION

The nonlinear coil is a crucial element of each ferroresonant circuit. Therefore performing a full scale supply voltage diagnostic is necessary [6, 11]. Literature survey delivers proofs of usage of basic  $u - i$  characteristics or quasi-hysteresis curves fitted by different functions [12 – 14]. In fact only measured hysteresis curve provides true information about iron core coil features and behavior [6]. The shape of a hysteresis curve significantly influences the ferroresonance phenomenon [15, 16]. What is no less important, this rule covers both major loop and various minor loops cases [17].

Recorded current waveform and previously computed flux linkage waveforms have been used to plot hysteresis loops of the coil. The characteristics of the nonlinear core coil used in analysed test circuit is shown in Fig. 3.

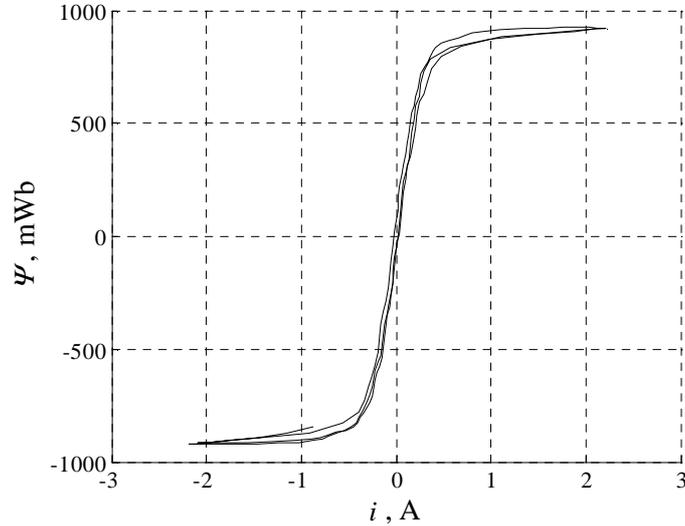


Fig. 3. Plot of measured flux linkage versus current (the hysteresis curve)

#### 4. VISUALISATION OF FERRORESONANCE

In literature one can find several tools dedicated to study the nonlinear and dynamic systems [18 – 21]. There are known methods based on bifurcation theory [22 – 25], where the evolution of the system solution is examined by changing the value of a control parameter. In a rather diagnostic approach the ferroresonance can be examined by using a spectral study method based on Fourier's analysis or a 2D/3D phase plane analysis supported by Poincaré maps [26 – 28]. The latter is used in this paper to differentiate between ferroresonance types. In the end it allows to correctly classify the ferroresonance that has occurred in the test circuit.

A system of differential equations is used to describe a nonlinear and dynamical test circuit [24, 29]. The set of equations depending on assumed mathematical model of nonlinear coil may be formulated:

$$\begin{cases} \frac{d\Psi}{dt} = f_1(\Psi, u_C, t) \\ \frac{du_C}{dt} = f_2(\Psi, u_C, t) \\ \frac{dt}{dt} = 1 \end{cases} \quad (2)$$

Such system of differential equations is solved numerically.

For the discussed series ferroresonance circuit, state space is a representation of time development of state variables  $u_C$  and  $\Psi$  [30, 31], which can be illustrat-

ed in a 2-dimensional or 3-dimensional plot (through trajectories). The phase trajectory displaying ferroresonance occurrence (transient state between two steady states) is presented in Fig. 4.

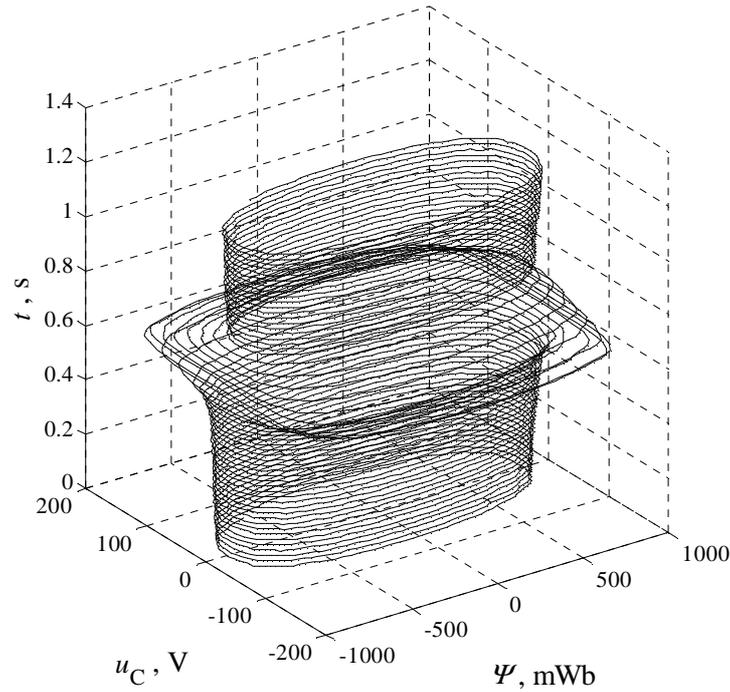


Fig. 4. Phase trajectory of the investigated system (where ferroresonance is observed)

In order to reduce dimensionality of information about changes of state variables one can analyze a two-dimensional plot. The whole three dimensional trajectory in the effect will be projected onto a 2D horizontal plane. The obtained state plane of the system is depicted in Fig. 5.

Presence of periodically driven voltage source and independent variables in the discussed system allows to generate another type of a three dimensional space. This space is an ordinary cylindrical coordinate system generated by rotating  $\Psi - u_C$  plane about a central vertical axis. The angular coordinate  $\omega_0 t$  represents the stage of the source waveform in its period  $T$ . The intersections of the trajectories with a plane at an arbitrarily selected angle are recorded. These intersections allow to generate what is called a Poincaré map. The trajectories on the 3-dimensional plane along with the points of the Poincaré map are presented in Fig. 6.

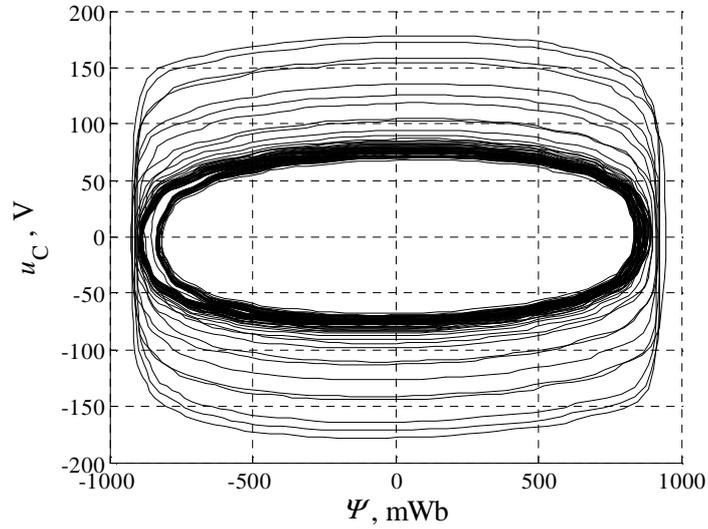


Fig. 5. Two dimensional phase plane of the investigated circuit obtained from previous (3-dimensional) phase space presented in Fig. 4

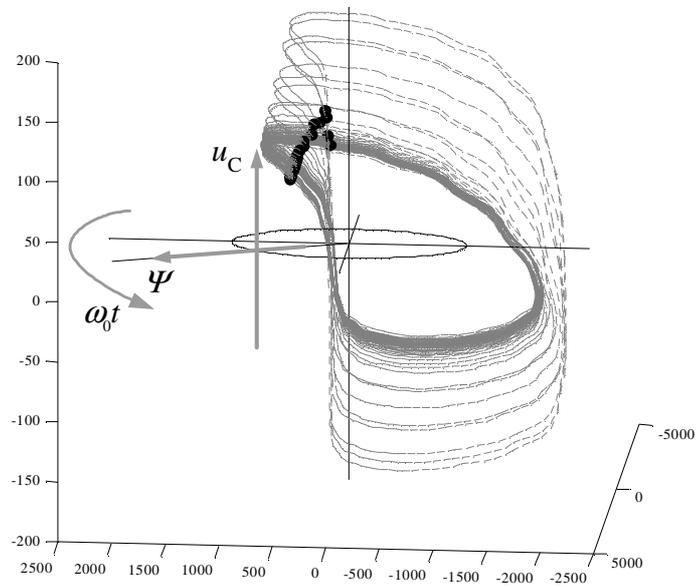


Fig. 6. Three dimensional representation of trajectory generated through the rotating  $\Psi - u_C$  plane for an examined ferroresonance transient state

The obtained Poincare map for the examined circuit is presented in Fig. 7.

A record of the intersection points (forming the Poincare map) along with the trajectory on a 2-dimensional plane is presented in Fig. 8.

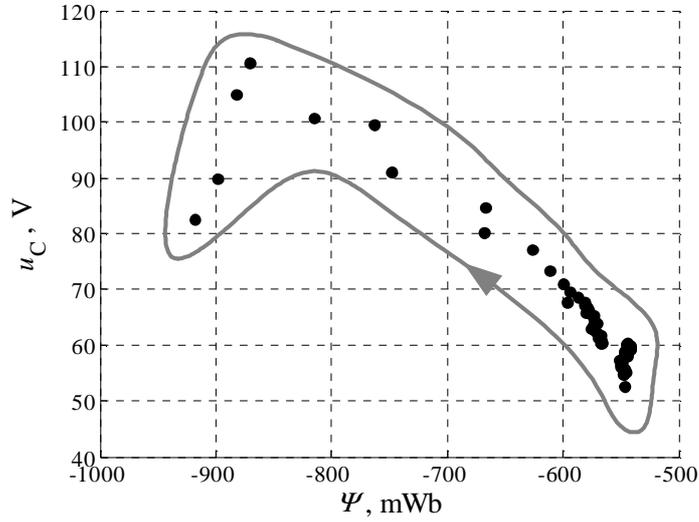


Fig. 7. Poincaré map for the studied trajectory

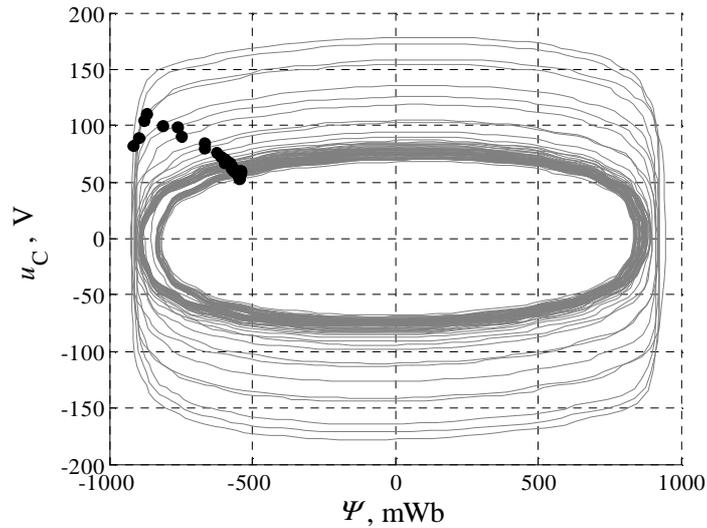


Fig. 8. Points of the Poincaré map and the phase trajectory of the system on a two-dimensional phase plane

The analysis of the features of an obtained Poincaré map can lead to determination of the type of ferroresonance, which in a practical system could lead to the identification of the source of the phenomenon.

A reliable ferroresonance mode classification has been given in the study [5]. The modes are:

- fundamental mode (one point on the Poincaré map can be observed),

- subharmonic mode (a single occurrence of a set of  $n$  points),
- quasi-periodic mode (several occurrences of a closed path of points),
- chaotic mode (many points gathered close to a single attractor).

Naturally - one observed ferroresonance can be potentially classified into two categories. The occurrence studied in this paper is one that can be classified as a quasi-periodic mode. The closed path of points on the Poincare maps (previously depicted in Fig. 7) can be observed. Several paths could be observed when observing Poincare maps in a larger time interval (e.g. from waveforms as in [6]).

## 5. CONCLUSIONS

The series ferroresonance circuit has been studied. All useful circuit quantities have been measured and computed and use for further analyses. Two of the obtained quantities have served as state variables of the considered system (i.e. nonlinear coil flux linkage and capacitor voltage). The obtained waveforms allows to illustrate 3D and 2D plots of the system phase trajectories. Further analysis of phase trajectories allowed to obtained a Poincare map. Finally, the Poincare map allowed to determined the type of the occurring ferroresonance.

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