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MODELS OF TRACTION STRAY CURRENTS INTERACTION WITH THE EARTH RETURN CIRCUITS

The paper presents methods for analyzing the impact of stray currents generated by DC electric traction on nearby earth-return circuits (pipelines). Various simulation models were developed and compared. First the current and potential of the rail were determined using two models. A model with distributed parameters and a model with lumped parameters consisting of chain-connected two-ports of the π type, for which the potential values in the nodes and currents in the branches were determined using the node potential method. The results obtained were used to further analyze the interaction between the railway rail and the pipeline. The pipeline model is presented in the form of a chain connection of π type two-ports. To determine the impact of stray currents on the pipeline, an analysis of the electric field in its vicinity was carried out. For this purpose, two methods have been developed. First, the analysis of the distribution of the scalar potential in the ground for a rail section of finite length was performed. Second method required taking into account the mutual conductivity of the two circuits (rail and pipeline), which in connection with the knowledge of the leakage current from the rail made it possible to determine the potential of the tested pipeline (using controlled voltage sources). The paper presents the implementation of methods and a comparison of the results obtained. The usefulness and applicability of the developed models for the analysis of the impact of stray currents from DC electric traction on earth-return circuits were also assessed.

KEY WORDS: stray currents, earth return circuits, interaction, simulation.

1. INTRODUCTION

Engineering practice often deals with problems connected with harmful effects (electrolytic corrosion) that direct current sources have on nearby earth-return circuits, e.g. underground metal pipelines, cables, etc. The stray currents from the d.c. rail-return circuit may flow into the earth and into the underground structure, returning to the rails or negative feeder taps in the vicinity of the substation or power plant. The general nature of the stray current problem is illustrated schematically in Fig. 1 [9, 10, 15].

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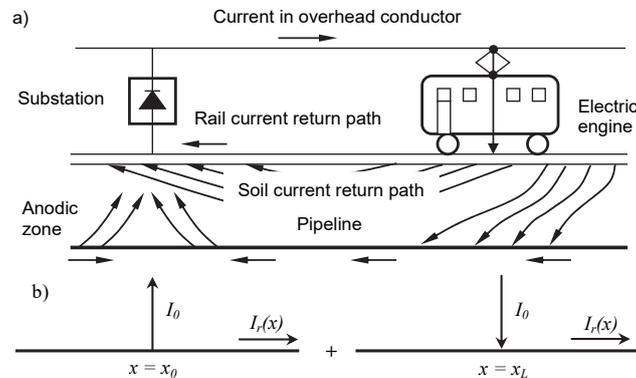


Fig. 1. Generation of stray currents: a) a railway system, b) railway system equivalents

Stray currents when entering or discharging metallic buried structures can cause damage in the form of structure coating disbonding in the area where the current enters and electrolytic corrosion in the area where it discharges (anodic zone). The extent of electrolytic damage is a function of the character of the metal and the amount of current and of time and soil conditions. Railway systems conventionally have the substation negative grounded to the rails. The system corrodes the rails remotely from the substation, whereas the foreign structures corrode near the substation, Fig. 1a.

The best approach to assessing stray currents interference is to determine the change in current density/potential on the structure subjected to the stray currents. Determining the response of the structure is not an easy task because it is a function of many factors: the location of the structure with respect to the electric flow field generated by the stray current source, the magnitude of the electric field, and the electrochemical response of the structure to the interference. In a row the outflow of stray currents into the ground depends on the properties of electric traction return circuits (the actual load of traction circuits i.e., the load of each electric locomotive, their number and position on the route, type and quality of rails and subgrade, and also the structure and conductivity of the surrounding environment, etc.). The positive potential deviation of the structure is a measurable effect which is used as electrolytic corrosion hazard criterion in standards and regulations of many countries. However, such measurements are usually post construction and thus may lead to an expensive retrofit program, in the case of undesirable results.

To predict the potential shift due to the stray current influence, calculation methods/tools can be used, especially at design stage of new traction lines or pipelines. The existing simulation models presented in the literature are mainly based on the deterministic approach, e.g. an analytical method of calculation basing on the complete field method of solution of the transmission-line prob-

lem. The analysis is applicable to any d.c. railway system in which tracks can be represented by a single earth return circuit (equivalent rail) with current (shunt) energization [9, 10, 15]. The method, similarly to the “field approach” – e.g. the Boundary Element Method [1, 2, 12], is an alternative to the approximate method in which the equivalent rail with current energization is modeled as a large multinode electrical equivalent circuit with lumped parameters. This circuit is a chain of basic circuits, which are equivalents of homogenous sections of the rail [3, 4, 7, 8, 13, 14].

The objective of the paper is to present problems of the modeling of stray currents effects generated by d.c. electrified railways. The analysis described in the paper may be useful in understanding effects on metal installation buried in the stochastic stray current area. The efficiency of the simulation program developed is demonstrated by illustrative calculations.

2. CURRENT AND POTENTIAL EXCITED IN A RAIL BY CURRENT ENERGIZATION

2.1. Solution for a rail modeled as a circuit with distributed parameters

The system shown in Fig. 1b may be applied directly by superposition in building up electrified railway system. In this system tracks are represented by a single conductor – equivalent to a rail continuously in contact with the earth through the track ballast. The conductor is energized with the currents I_0 and $(-I_0)$ by a feeder station and a load at points $x = x_0$ and $x = x_L$, respectively.

It should be noted that the basic model can be applied directly by superposition if there is a number of substations and loads to be considered.

The starting point for the analytical solution for current and potential along an equivalent rail located along the x – axis of the Cartesian coordinate system is, according to the multi-conductor line theory, the system of linear differential equations:

$$\begin{aligned} -\frac{dV_r(x)}{dx} &= Z_r I_r(x) - E_s(x) \\ -\frac{dI_r(x)}{dx} &= Y_r V_r(x) - J_s(x) \end{aligned} \tag{1}$$

where V_r denotes the rail potential, I_r – the rail current, Z_r – the longitudinal impedance (resistance) per unit length (p.u.l.), Y_r – the p.u.l. shunt admittance (conductance), and E_s and J_s are the p.u.l. external sources (longitudinal and

shunt, respectively) driving the homogeneous line. The details of the circuit with earth return parameters can be found in the literature, e.g. [9, 10, 15].

If the equivalent rail is infinite in the length and energized with the current I_0 by a substation at point $x = x_0$, the solution of the eqns (1) for the current along the rail, taking into account the boundary conditions:

$$I_r(x_0^-) = \frac{I_0}{2}, \quad I_r(x_0^+) = -\frac{I_0}{2} \quad (2)$$

where $I_r(x_0^-)$ and $I_r(x_0^+)$ denote the left-hand and right-hand limits of the function $I_r(x)$ when x approaches to x_0 , is given in the form [9, 10, 15]

$$I_r(x) = -\text{sign}(x - x_0) \frac{I_0}{2} e^{-\Gamma|x-x_0|} \quad (3)$$

where

$$\text{sign}(x - x_0) = \begin{cases} -1 & \text{when } x - x_0 < 0 \\ 1 & \text{when } x - x_0 > 0 \end{cases} \quad (4)$$

and Γ is the propagation constant.

Potential along the equivalent rail can be calculate from the relationship:

$$V_r(x) = -\frac{1}{Y_r} \frac{dI_r(x)}{dx} \quad (5)$$

thus taking into account formulas (3) and (5)

$$V_r(x) = -Z_0 \frac{I_0}{2} e^{-\Gamma|x-x_0|} \quad (6)$$

where: Z_0 – characteristic impedance of the equivalent rail. The details of the parameters of the rails and the equivalent rail can be found in literature, e.g. [5, 6, 11].

For the case of current energization of the rail by a vehicle at the point $x = x_L$ (Fig. 1b), currents and potentials are calculated from the equations (3) and (5) with $I_0 = -I_0$ and $x_0 = x_L$, respectively.

Consider next the case of a finite rail extending from $x = x_1$ to $x = x_2$. The rail is energized with the current I_0 at $x = x_0$ and is open circuited on both ends. The current along the rail can be now determined from the following expression:

$$I_r(x) = -\text{sign}(x - x_0) \frac{I_0}{2} e^{-\Gamma|x-x_0|} + A_1 e^{-\Gamma x} + B_1 e^{\Gamma x} \quad (7)$$

where A_l and B_l are constants which are to determine from the boundary conditions.

Taking into account that:

$$I_r(x_1) = I_r(x_2) = 0 \quad (8)$$

the constants A_l and B_l become:

$$A_l = -\frac{I_0}{2} \frac{ch\Gamma(x_2 - x_0)}{sh\Gamma L} e^{\Gamma x_1}, \quad B_l = \frac{I_0}{2} \frac{ch\Gamma(x_0 - x_1)}{sh\Gamma L} e^{-\Gamma x_2} \quad (9)$$

where $L = x_2 - x_1$ denotes the rail length.

It is easy to show that for the case of finite length rail which is energized at the point x_L by a load current ($-I_0$), the rail current becomes

$$I_r(x) = sign(x - x_L) \frac{I_0}{2} e^{-\Gamma|x-x_L|} + A_2 e^{-\Gamma x} + B_2 e^{\Gamma x} \quad (10)$$

and

$$A_2 = \frac{I_0}{2} \frac{ch\Gamma(x_2 - x_L)}{sh\Gamma L} e^{\Gamma x_1}, \quad B_2 = -\frac{I_0}{2} \frac{ch\Gamma(x_L - x_1)}{sh\Gamma L} e^{-\Gamma x_2} \quad (11)$$

It should be pointed out, that for the case of other kind of the boundary conditions, e.g. defined by impedances of finite value at rail both ends, the constants can be evaluated in similar way.

2.2. Solution for a rail modeled as a circuit with lumped parameters

Assuming a segment of the length dl of the equivalent rail to be homogeneous (e.g. $Z_r, Y_r = \text{const.}$), it is possible to model the circuit by a π – two port, as shown in Figure 2 [3, 4, 8], with the series impedance and the shunt admittance

$$Z_r = Z_0 \sinh(\Gamma dl), \quad Y_r = \frac{2 \tanh(\Gamma dl / 2)}{Z_0} \quad (12)$$

where: Z_0 – characteristic impedance, Γ – propagation constant.

The whole rail length can be subdivided into elementary cells which may have different lengths or different specific parameters.

If the rail is subjected to the external sources, the passive model has to be completed by the active elements. This leads to a circuit representation for the current/shunt energization (substation and loads) of the equivalent rail, Fig. 2.

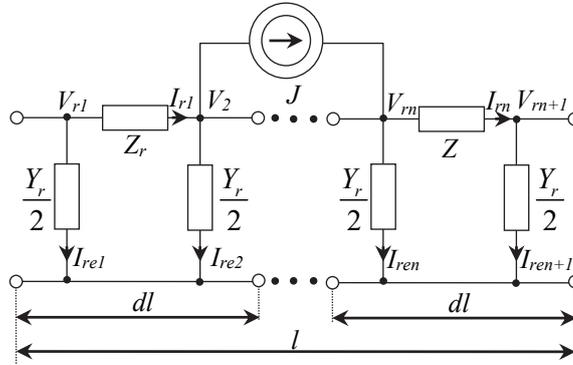


Fig. 2. π - two port model of the rail with a current source representing the current energization

After being divided into sections the equivalent rail can be composed of such basic two-ports which define the nodes and branches of the network model, which is well suited for computer - aided circuit analysis using simulation programs. The number of subdivisions of the rail can theoretically be as large as required, according to the wanted degree of discrimination in the potential and current computation.

One of the method that can be used to calculate rail potential and current is the nodal analysis. The matrix notation for this method for circuit from Fig. 2 shows equation (13):

$$\begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_n \end{bmatrix} = \begin{bmatrix} \frac{Y_r}{2} + \frac{1}{Z_r} & -\frac{1}{Z_r} & \cdots & 0 \\ -\frac{1}{Z_r} & Y_r + \frac{2}{Z_r} & \cdots & 0 \\ \vdots & \vdots & \ddots & -\frac{1}{Z_r} \\ 0 & 0 & -\frac{1}{Z_r} & \frac{Y_r}{2} + \frac{1}{Z_r} \end{bmatrix} \cdot \begin{bmatrix} V_{r1} \\ V_{r2} \\ \vdots \\ V_{rn} \end{bmatrix} \quad (13)$$

where: J is current of energization (source current), Z and Y are electrical parameters of rail (there are elements of matrix with self and mutual admittances), V_r is potential of rail in the nodes.

To calculate rail potential transformation of the relationship (13) to (14) is necessary:

$$\begin{bmatrix} V_{r1} \\ V_{r2} \\ \vdots \\ V_m \end{bmatrix} = \begin{bmatrix} \frac{Y_r + 1}{2Z_r} & \frac{-1}{Z_r} & \dots & 0 \\ \frac{-1}{Z_r} & Y_r + \frac{2}{Z_r} & \dots & 0 \\ \vdots & \vdots & \ddots & \frac{-1}{Z_r} \\ 0 & 0 & \frac{-1}{Z_r} & \frac{Y_r + 1}{2Z_r} \end{bmatrix}^{-1} \cdot \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_n \end{bmatrix} \quad (14)$$

The current in the rail is calculated using equation:

$$I_m = \frac{V_n - V_{n+1}}{Z_r} \quad (15)$$

2.3. Example (1) of calculation

Two following cases have been proposed to present the algorithm developed. The rail is modeled by equivalent finite long earth return circuit with parameters $Z = 0.02 \Omega/\text{km}$ and $Y = 0.76 \text{ S/km}$. Substation is located at point $x_s = 7.5 \text{ km}$ of selected 20 km rail section. Vehicle is located at point $x_L = 12.5 \text{ km}$. Soil has conductivity 0.01 S/m . Value of station and vehicle current is equal $I_0 = 1 \text{ A}$. The results of the rail potential and current (V_r, I_r – model with distributed parameters, V_r, I_r – model with lumped parameters) calculations are presented in Fig. 3 and 4.

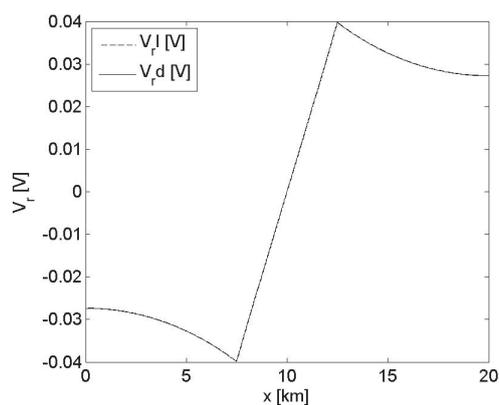


Fig. 3. Comparison of rail potentials for two methods

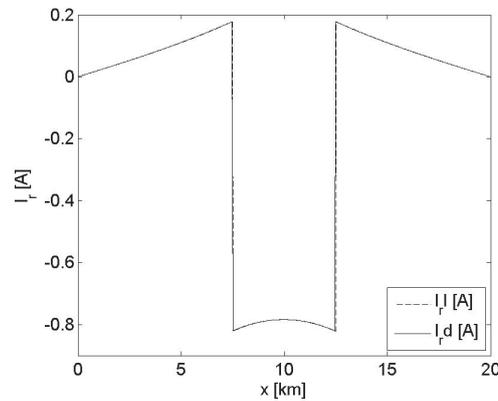


Fig. 4. Comparison of rail current for two methods

3. CALCULATION OF POTENTIAL OF A PIPELINE LOCATED IN STRAY CURRENT AREA

3.1. Scalar potential in the earth due to current in the equivalent rail

The knowledge of the earth potential of the electric flow field in the vicinity of the tracks is required for the evaluation of stray currents effects on nearby structures. The potential (primary potential) can be obtained by the technique used in the earth return circuit theory, when the conductor with earth return carries a longitudinal current [9, 10, 15]. The basic circuit for the calculation of the earth potential is shown in Fig. 5.

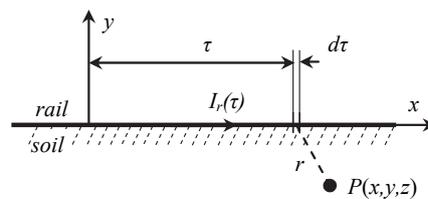


Fig. 5. Equivalent rail with longitudinal current flow on the earth surface

The equivalent rail is placed on the earth surface and is carrying the longitudinal current $I_r(x)$ which flows in the positive direction of the x axis lying along the rail. The rail can be regarded as a set of current elements of length $d\tau$. From each element an elementary leakage current $(-dI_r(\tau)/d\tau)$ flows into the earth with the conductivity γ , producing the elementary scalar potential. In the observation point $P(x,y,z)$ the scalar potential can be determined from the expression:

$$dV_e^0(P) = -\frac{I}{2\pi\gamma r} \frac{dI_r(\tau)}{d\tau} \quad (16)$$

where r is the distance from the current element (source point) to the observation point.

For the case of finite length equivalent rail, located in the xy plane ($y = 0$, $z = 0$), the scalar potential in the earth becomes:

$$V_e^0(x, y, z) = \frac{I}{2\pi\gamma} \int_L \frac{-dI_r(\tau)/d\tau}{\sqrt{(x-\tau)^2 + y^2 + z^2}} d\tau \quad (17)$$

If a finite rail extending from $x = x_1$ to $x = x_2$, is energized with the current I_0 at $x = x_0$ and open circuited on both ends, the current along the rail is described by eqn. (7). Thus the scalar potential can be determined from the following expression:

$$V_e^0(P) = \frac{I_0\Gamma}{4\pi\gamma} \left[-e^{-\Gamma x_0} \int_{x_1}^{x_0} \frac{e^{\Gamma\tau}}{\sqrt{(x-\tau)^2 + y^2 + z^2}} d\tau + \right. \\ \left. -e^{\Gamma x_0} \int_{x_0}^{x_2} \frac{e^{-\Gamma\tau}}{\sqrt{(x-\tau)^2 + y^2 + z^2}} d\tau + \right. \\ \left. -\frac{ch\Gamma(x_2 - x_0)}{sh\Gamma L} e^{\Gamma x_1} \int_{x_1}^{x_2} \frac{e^{-\Gamma\tau}}{\sqrt{(x-\tau)^2 + y^2 + z^2}} d\tau + \right. \\ \left. -\frac{ch\Gamma(x_0 - x_1)}{sh\Gamma L} e^{-\Gamma x_2} \int_{x_1}^{x_2} \frac{e^{\Gamma\tau}}{\sqrt{(x-\tau)^2 + y^2 + z^2}} d\tau \right] \quad (18)$$

For the case of current energization of the finite length rail by a vehicle at the point $x = x_L$ the scalar potential is calculated from the eqn. (15) with $I_0 = -I_0$ and $x_0 = x_L$, respectively.

The total earth potential in the observation point P results from the superposition and can be numerically solved.

3.2. Model of pipeline energized by controlled voltage source

To analyze the current which flows in a pipeline it is necessary to know the parameters of electric field which causes energization of the pipeline. The method described in point 3.1 can be used. The scalar potential of the electric field can be modeled using controlled voltage sources (V_e^0). It is shown in Fig. 6a.

However the solution of equations used in this method is complicated. It increases calculation time and special algorithms must be applied. An alternative to this method is to determine the pipeline energization parameters in a different way. One of that way is to determine the coupling of the pipeline with the rail using mutual conductance between both earth-return circuits. On this basis and knowing values of the leakage current from the rail, it is possible to determine the voltage value of the controlled voltage source (V_p – Fig. 6b) supplying the pipeline:

$$V_{pi} = \frac{I_{ei}}{G_{rp}} \cdot dl \quad (19)$$

where: I_e - leakage current from the rail, G_{rp} - mutual conductance per unit length between earth-return circuits given by equation:

$$G_{rp}^{-1} = \frac{1}{\pi\gamma} \ln \frac{1.12}{s_{rp} \sqrt{\Gamma_r \Gamma_p}} \quad (20)$$

where: Γ_r - rail propagation constant, Γ_p - pipeline propagation constant, γ - earth conductivity, s_{rp} - distance:

$$s_{rp} = \sqrt{(d_r - d_p)^2 + a_{rp}^2} \quad (21)$$

where: d_p - burial depth of a pipeline, d_r - burial depth of a rail, a_{rp} - horizontal distance between earth-return circuits.

The leakage current from the rail can be calculated from the relation:

$$I_{ei} = V_{ri} \cdot Y_r \cdot dl \quad (22)$$

where: V_{ri} - rail potential, Y_r - rail admittance per unit length, dl - elementary length of rail.

Equivalent circuits represented pipeline with different controlled voltage sources are shown in Fig. 6.

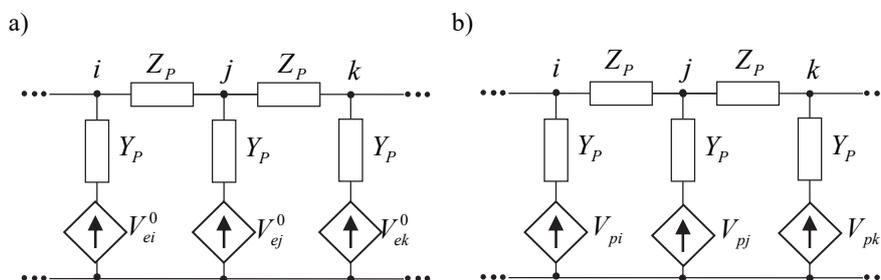


Fig. 6. a) π - two port model of the pipeline with a controlled voltage sources representing the pipeline energization by earth scalar potential, b) π - two port model of the pipeline with a controlled voltage sources representing conductive energization of the pipeline

3.3. Example (2) of calculation

The simulation model is the same as in point 2.3. The pipeline with the diameter 355.6 mm has a length of 20 km, and its electrical parameters are: $Z_p = 0.02 \Omega/\text{km}$, $Y_p = 0.011 \text{ S}/\text{km}$. The pipeline is buried in the soil with conductivity 0.01 S/m at 1.0 m depth centrally under the rail. The pipeline is open circuited at its both ends. Value of station and vehicle current is equal $I_0 = 1 \text{ A}$. The results of simulation are shown in Fig. 7.

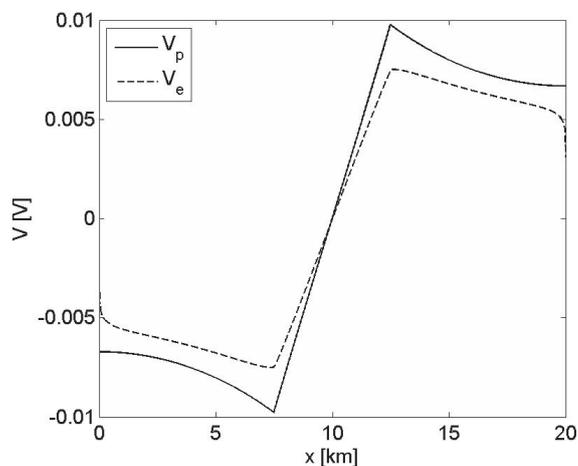


Fig. 7. Potential distributions along the pipeline (V_p – potential of controlled voltage sources representing conductive energization of the pipeline) and the earth potential (V_e - potential controlled voltage sources representing the pipeline energization by earth scalar potential)

FINAL REMARKS

The methods presented in the paper are used to analyze stray current phenomena in traction systems and their interaction with other earth return circuits (eg. metallic pipelines). The results of comparison for methods used to calculation of rail potential and current are very similar. The difference between results is less than 1%. This confirms the correct operation of both models. The second calculation example compares two methods used to determine the impact of the electric field from DC traction on the underground pipeline. In this case, the results of the comparison of the two methods used are also acceptable. It needs to be highlighted that appliance the models with lumped parameters decreases the time of calculations.

The approach used enables any physical interpretation of the phenomena being simulated. The formulas obtained in the paper require numerical integration which can be performed by the use of freely available tools.

The analysis described in the paper may be useful in understanding effects on metal installation buried in the stray current area. The simulation models presented can be especially useful in the design stage of new earth return circuit buried in the stray current area, when frequent alterations are made as the design progresses.

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